St Anne's Multiplication Calculation Policy

Date approved:	19th February, 2024
Date of review:	Summer Term 2025

Rationale:

This policy lays out the expectations for written calculations using multiplication and has been created to support the teaching of a mastery approach to mathematics in line with the National Curriculum and the White Rose scheme, which forms the framework of our curriculum through its long- and medium-term planning outline and small steps. This is underpinned by the use of models and images that support conceptual understanding and this policy promotes a range of representations to be used across EYFS, KS1 and KS2.

A Mastery Approach:

A mastery approach to learning involves the following five "big ideas" of effective maths teaching:

Coherence -	a coherent learning progression offering deep and connected understanding
Representation and Structure	concrete, pictorial and abstract representations are carefully structured to help pupils "see the maths"
Mathematical Thinking -	looking for patterns and relationships, making connections, conjecturing, reasoning and generalising, communicating ideas using precise vocabulary
Fluency - - -	efficient, accurate recall of key number facts and procedures, allowing pupils to move between different contexts and representations, choosing strategies
Variation - - -	conceptual variation presents different representations of key features, while procedural variation presents different ways of proceeding through the learning journey (via scaffolding and support, etc)

Concrete – Pictorial – Abstract:

Mathematical understanding is developed through use of representations that are initially concrete (e.g. counters, multilink cubes, dienes, etc), and then pictorial (e.g. part-whole models, place value columns with images of counters in them, etc) to then facilitate abstract working (e.g. formal written methods).

This policy is a guide through an appropriate progression of representations. If at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate. As children move through the different stages, representations should be modelled alongside each other to ensure a secure understanding is maintained. Children should only move onto the abstract method when they have a secure understanding of the concept through an appropriate concrete and pictorial representation. This policy should be used in conjunction with the St Anne's Mathematics policy and St Anne's Mental Calculation policy. Teachers are also encouraged to refer to the NCETM Ready-To-Progress Criteria resources in ascertaining when children are ready to move on to new learning.

Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of recommendations and effective practice guidance aimed at informing and enhancing teaching across all year groups. Many of these ideas come from the NCETM's Calculation Guidance document

(published October 2015) and the White Rose Calculation Guidance (published in the academic year 2020-2021), which is intended to sit alongside a school's calculation policy.

Please note that while this list of representations is illustrative of the representations children will be exposed to and work with, it is not intended to be exhaustive and children will also encounter other representation methodologies that are not listed here.

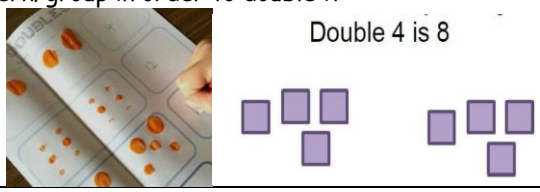
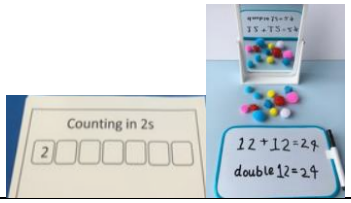
Vocabulary:

Children will continually recap vocabulary learned in previous years to ensure that their understanding and usage of the terminology is fully developed, broad and specific in application. Vocabulary from previous years is included in each year group's columns in black, while new vocabulary that may not have been previously encountered is in green. Teachers are encouraged to check this list of vocabulary at the beginning and end of a relevant unit to ensure that they are modelling the full breadth and depth of vocabulary to the children, and that the children are using it in their verbal and written responses accurately and confidently.

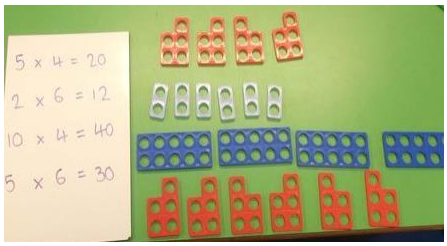
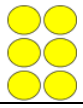
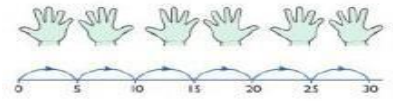
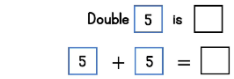
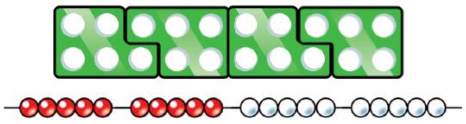
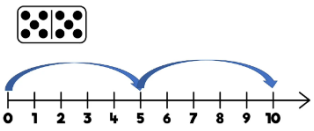
Please see appendix 4 for notes on precise vocabulary, and for a comprehensive glossary, please see the separate document "NCETM Maths Glossary KS1-KS3" which is saved in PDF format with our calculation policies in the shared area.

Contents:

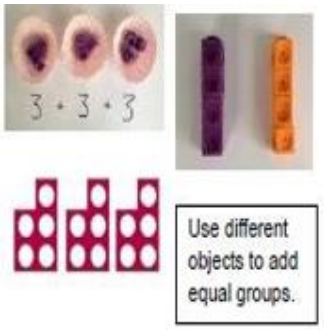
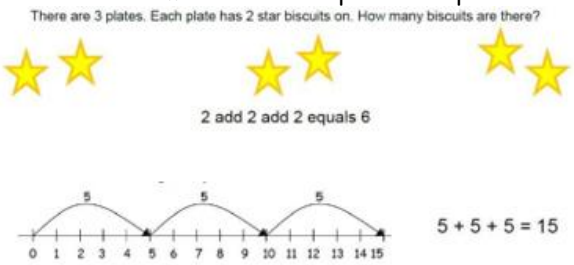
Multiplication in Reception

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Jump along a number line in steps of … Look at patterns and counting. Grouping objects, counting groups of the same size. Double. Talk about odds and evens.	odd, even, double, groups of, ones, twos, tens, count in …	**Concrete:** Children use Numicon, dominoes and counters to support counting in jumps and doubling:  **Pictorial:** Children use various representations of numbers using shapes, dots, etc to repeat a pattern/group in order to double it:  **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children use number tracks and addition number sentences to work with doubling: 

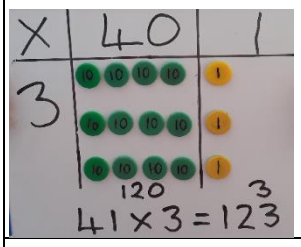
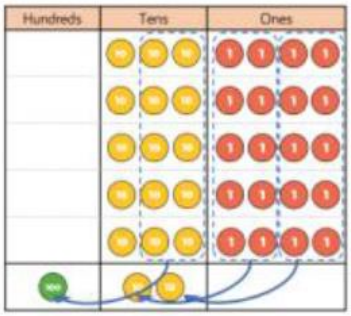
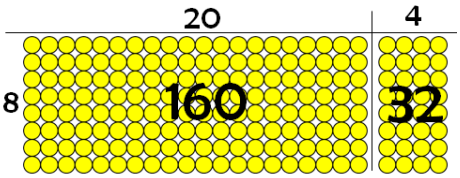
Multiplication in Year 1

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays, with the support of the teacher.	odd, even, double, groups of, ones, twos, **threes, fives,** tens, count in …, **(forwards from/backwards from), how many times, lots of, groups of, once, twice, ___ times, multiple/s of ___, times, multiply, multiplied by, repeated addition, array, row, column**	**Concrete:** Children use various objects as representations to make groups and arrays, as well as making groups on bead strings:   Children will also use Numicon to represent multiplication (right), and counters laid out in arrays (below) to reinforce the idea of rows and columns (also commutativity: 3 rows of 2 = 3 columns of 2).   **Pictorial:** Children will count on number lines to reinforce repeated addition:  Children will also use Numicon representations and bead string images as with the concrete representations above, as well as dominoes:    Pictures laid out in arrays support one-step word problems:  There are _____ trays of cakes. There are _____ cakes on each tray. There are _____ cakes altogether. **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children show multiplication using simple repeated addition number sentences such as 2 + 2 + 2 = 6, or for doubling 5 + 5 = 10, for example.

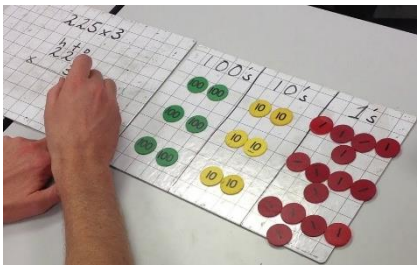
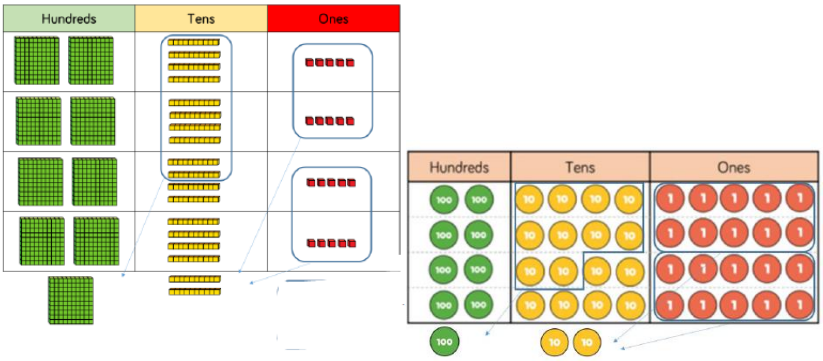
Multiplication in Year 2

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×) and equals (=) signs. Show that multiplication of two numbers can be done in any order (commutative). Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication facts, including solving problems in contexts. **Times Tables:** By the end of year 2, children should know their 2x, 5x and 10x tables and associated multiplication and division facts out of order.	odd, even, double, groups of, ones, twos, threes, fives, tens, count in …, (forwards from/backwards from), how many times, lots of, groups of, once, twice, ___ times, multiple/s of __, times, multiply, multiplied by, repeated addition, array, row, column, **commutative law,**	**Concrete:** Children use various objects to create equal groups, reinforcing repeated addition as a simple methodology to follow to find the product:  **Pictorial:** Children use number lines to represent repeated addition:  Children also use money (both as concrete resources and as pictorial representations) to form equal groups and use their times tables to calculate products:  **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children continue to show multiplication using simple repeated addition number sentences such as 2 + 2 + 2 = 6, or for doubling 5 + 5 = 10, for example, before moving into writing the related facts as multiplication sentences such as 2 x 3 = 6 and 3 x 2 = 6, and 2 x 5 = 10 and 5 x 2 = 10, in the above examples. Children also encounter word problems such as: *One bag holds 5 apples. How many apples do 4 bags hold?*

Multiplication in Year 3

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Recall and use multiplication facts for the 3, 4 and 8 multiplication tables. Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for 2-digit numbers times 1-digit numbers, using mental and progressing to formal written methods. Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which 'n' objects are connected to 'm' objects. **Times Tables:** By the end of year 3, children should know their 2x, 3x, 4x, 5x, 8x and 10x tables and associated multiplication and division facts out of order.	odd, even, double, groups of, ones, twos, threes, fives, tens, count in …, (forwards from/backwards from), how many times, lots of, groups of, once, twice, ___ times, multiple/s of __, times, multiply, multiplied by, repeated addition, array, row, column, commutative law, **multiplicand, multiplier, product, scale up, regroup, exchange,**	**Concrete:** Children look at arrays (right) in different orientations to see the commutative law as it relates to the array. For example: 4 x 6 = 24 and 6 x 4 = 24   Concrete resources can also be used when learning grid method (left). **Pictorial:** Children use arrays to show multiplication using partitioning (below), before moving onto using place value counters in place value columns to show how the process can involve regrouping/exchanging:   **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Children will use grid method as a continuation of the above arrays using partitioning to find products (right). x | 50 | 4 6 | 300 | 24 300 + 24 324 ***Note: Positive integer scaling problems*** *are where two things are in proportion, so affecting one will affect the other in the same way. For example:* *If 3 apples cost 50p, then 6 apples cost £1 (we scaled up by 2).* ***Correspondence problems*** *are where two sets of objects or values are related, and all possible combinations must be found. For example:* *I have 3 hats and 4 coats. How many different outfits can I make? or* *On the planet Vuv there are Zios (which have 3 legs) and Zepts (which have 7 legs). Nico the explorer counted 7 Zepts and a Zio. How many legs do they have altogether?*

Multiplication in Year 4

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Recall multiplication facts for multiplication tables up to 12 × 12. Use place value, known and derived facts to multiply mentally, including multiplying by 0 and 1, and multiplying together 3 numbers. Recognise and use factor pairs and commutativity in mental calculations. Multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout. Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as when "n" objects are connected to "m" objects. **Times Tables:** By the end of year 4, children should know all of their times tables and associated multiplication and division facts up to 12x12 out of order.	odd, even, double, groups of, ones, twos, threes, fives, tens, count in …, (forwards from/backwards from), how many times, lots of, groups of, once, twice, ___ times, multiple/s of ___, times, multiply, multiplied by, repeated addition, array, row, column, commutative law, multiplicand, multiplier, product, scale up, regroup, exchange, **multiplication facts, division facts, inverse, derive**	**Concrete:** Children will use place value counters or base 10 to form the multiplicand, and do so the appropriate number of times as dictated by the multiplier. In this example, 225 has been made 3 times to work out 225x3:  Note: Where possible, it is good practice to also show the process in an abstract way simultaneously, so that children can increase exposure to the more efficient compact column method as they build their understanding using concrete resources. **Pictorial:** Children use pictorial representations of the same concept from above, in order to show the process of regrouping/exchanging which takes place:  **Abstract:** ***See Appendix 1 for further guidance on best practice.*** Initially, children will use grid method (right) to explore multiplication as in year 3. 5 x 123 = 100 20 3 (5 x 100) + (5 x 20) + (5 x 3) |	100	20	3
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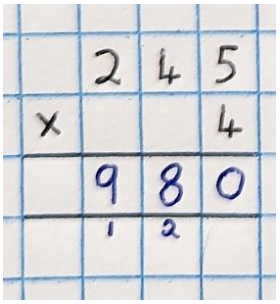
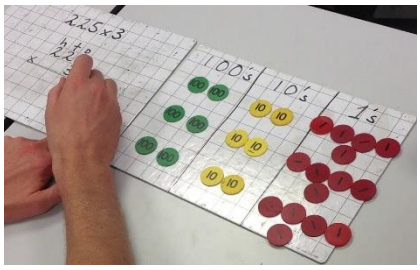
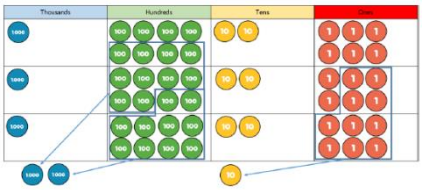
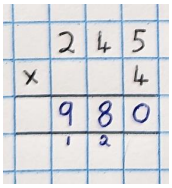
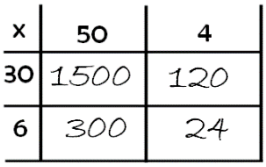
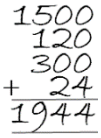
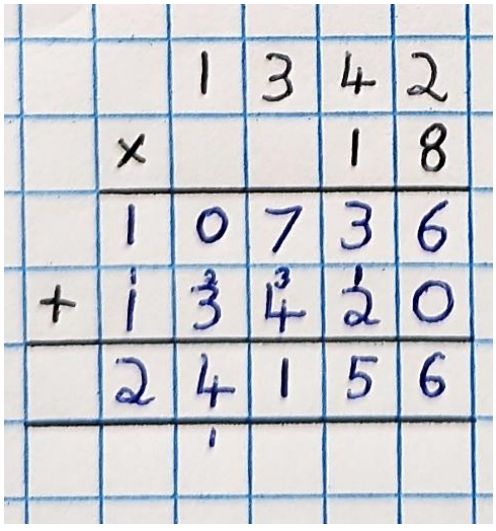
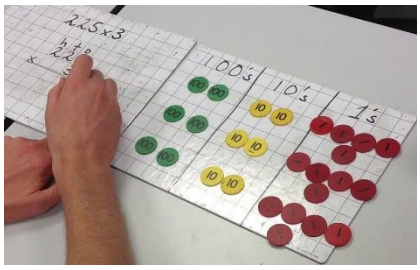
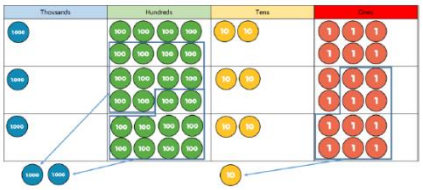
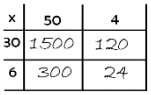
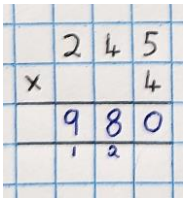
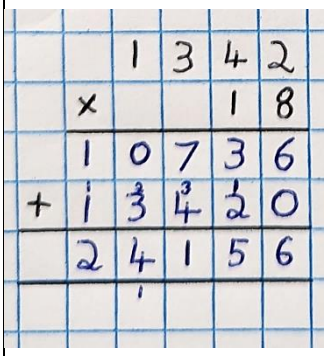
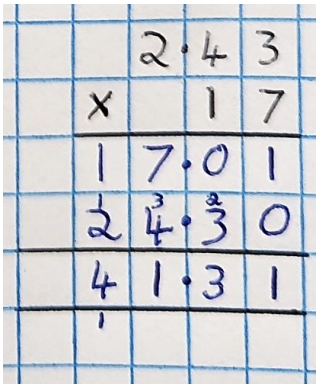
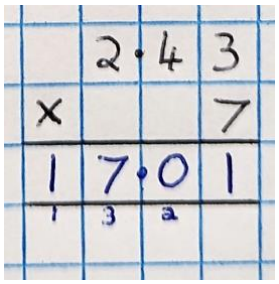
5	500	100	15	 500 + 100 + 15 = Children will then use a formal columnar algorithm, initially introduced via appropriate concrete and/or pictorial representations in place value columns, to multiply multiplicands of up to 3 digits by 1-digit multipliers. Children should not begin to use a compact column method before having a secure understanding of the process of multiplication, arrays, partitioning, exchanging/regrouping, and using place value columns.  Children should be taught when it is best to use a mental method for straightforward calculations, when to use number lines, grid method or other informal jottings for relatively straightforward ones, and when to use column methods for the more complex calculations that require it. ***See note in Year 3's Abstract section for explanation and examples of integer scaling problems and correspondence problems.***

Multiplication in Year 5

National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers. Multiply numbers mentally drawing upon known facts. Multiply whole numbers and those involving decimals by 10, 100 and 1,000. Solve problems involving multiplication including using their knowledge of factors and multiples, squares and cubes. Solve problems involving multiplication, including understanding the meaning of the equals sign. Solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates. Solve problems involving number up to 3 decimal places. **Times Tables:** Children should continue to practise times tables and associated multiplication and division facts out of order in year 5.	odd, even, double, groups of, ones, twos, threes, fives, tens, count in …, (forwards from/backwards from), how many times, lots of, groups of, once, twice, ___ times, multiple/s of ___, times, multiply, multiplied by, repeated addition, array, row, column, commutative law, multiplicand, multiplier, product, scale up, regroup, exchange, multiplication facts, division facts, inverse, derive, **most efficient method**	**Concrete:** Children will use place value counters or base 10 to form the multiplicand, and do so the appropriate number of times as dictated by the multiplier. In this example, 225 has been made 3 times to work out 225x3:  Note: Where possible, it is good practice to also show the process in an abstract way simultaneously, so that children can increase exposure to the more efficient compact column method as they build their understanding using concrete resources.
		Pictorial: Children use pictorial representations of the same concept from above, in order to show the process of regrouping/exchanging which takes place: 
		Abstract: ***See Appendix 1 for further guidance on best practice.***  Short multiplication with a 1-digit multiplier will be completed using the same method as in Year 4 (right). Initially, children will use grid method (right) to explore long multiplication.   Children will then use a formal columnar algorithm, initially introduced via appropriate concrete and/or pictorial representations in place value columns, to multiply multiplicands of up to 4 digits by 2-digit multipliers (below, right).  Children should not begin to use a compact column method before having a secure understanding of the process of multiplication, arrays, partitioning, exchanging/regrouping, and using place value columns. Children should be taught when it is best to use a mental method for straightforward calculations, when to use number lines, grid method or other informal jottings for relatively straightforward ones, and when to use column methods for the more complex calculations that require it.

Multiplication in Year 6

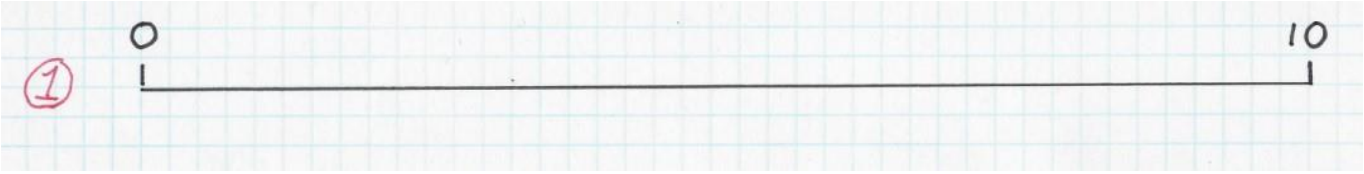
National Curriculum Objectives and Strategies	Vocabulary	Example Representation Methodologies
Children will… Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. Multiply one-digit numbers with up to two decimal places by whole numbers. **Times Tables:** Children should continue to practise times tables and associated multiplication and division facts out of order in year 6.	odd, even, double, groups of, ones, twos, threes, fives, tens, count in …, (forwards from/backwards from), how many times, lots of, groups of, once, twice, ___ times, multiple/s of __, times, multiply, multiplied by, repeated addition, array, row, column, commutative law, multiplicand, multiplier, product, scale up, regroup, exchange, multiplication facts, division facts, inverse, derive, most efficient method, **order of operations**	**Concrete:** Children will use place value counters or base 10 to form the multiplicand, and do so the appropriate number of times as dictated by the multiplier. In this example, 225 has been made 3 times to work out 225x3:  Note: Where possible, it is good practice to also show the process in an abstract way simultaneously, so that children can increase exposure to the more efficient compact column method as they build their understanding using concrete resources. **Pictorial:** Children use pictorial representations of the same concept from above, in order to show the process of regrouping/exchanging which takes place:  **Abstract:** ***See Appendix 1 for further guidance on best practice.*** As in Years 4 and 5, short multiplication with a 1-digit multiplier will be completed using the formal columnar method (right). Children who struggle with the concept can use grid method (left) to support long multiplication.     Children will then use a formal columnar algorithm, initially introduced via appropriate concrete and/or pictorial representations in place value columns, to multiply multiplicands of up to 4 digits by 2-digit multipliers (left). They will also use this formal method for multiplicands with up to two decimal places (below).   Children should not begin to use a compact column method before having a secure understanding of the process of multiplication, arrays, partitioning, exchanging/regrouping, and using place value columns. Children should be taught when it is best to use a mental method for straightforward calculations, when to use number lines, grid method or other informal jottings for relatively straightforward ones, and when to use column methods for the more complex calculations that require it.

Appendix 1 - Notes on Best Practice with Written Methods

NCETM Guidance on Number Lines

(From https://www.ncetm.org.uk/features/five-tips-for-using-number-lines-in-key-stage-1/)

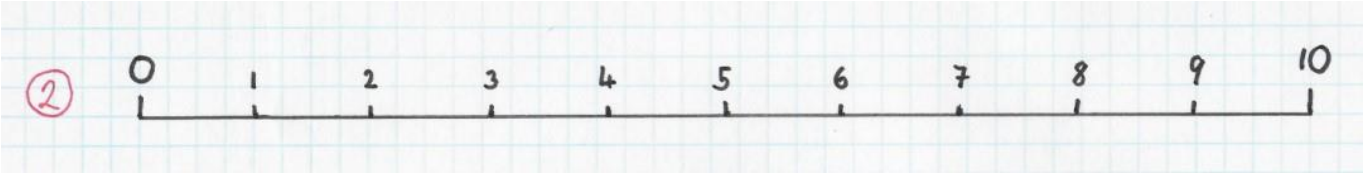
1. Finding the midpoint on a blank number line



Our first number line is a straight line with zero at one end and ten at the other. Could your pupils place the numbers one to nine on the straight line? It is likely numbers will be crowded together at either end of the line because the children have yet to develop the spatial reasoning skills needed. Asking pupils to find the midpoint (before asking them to place all the numbers) allows us to draw the children's attention to the fact that numbers are evenly spaced and to reinforce the concept of five as a midpoint.

A number line, or strip of paper folded in half, can be used to give children the opportunity to find the midpoint. Children can also create lines of different lengths on the playground with chalk or skipping ropes, and practise placing the class teddy on the midpoint. This activity provides a great opportunity to develop classroom talk and spatial language, such as 'in the middle' and 'equally spaced'.

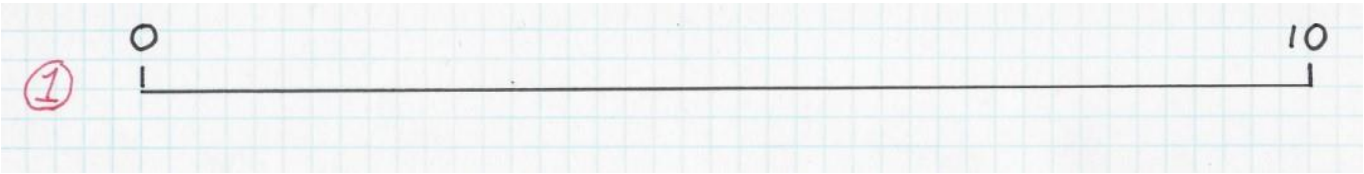
2. Reasoning with a completed zero to ten number line



You could start this activity by revisiting the idea of the midpoint and ask children how they know what the midpoint is, providing a further opportunity for children to practise spatial language. However, the beauty of this number line is the opportunities it provides to fully explore the linear nature of the number system and to reason about the location of numbers within it.

One way to start this exploration is through games of true or false using the inequality symbols; for example, seven is greater than eight. Can the children prove their answer using the number line? Another game that could be played is Guess My Number. Can children use clues about a number's location on the number line to find the number? These games bring the children back to the spatial language of 'greater than' and 'less than'. They are asked to justify their answers, whilst their attention is also drawn to where these numbers fit on the number line.

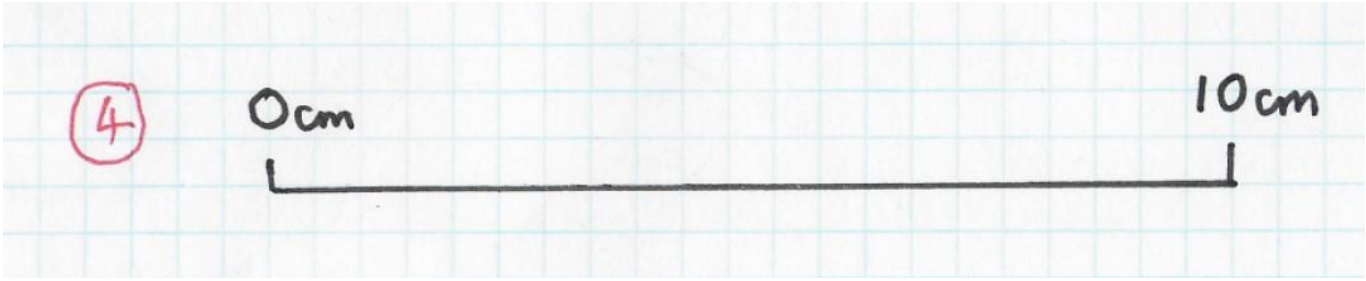
3. Placing numbers on a blank number line



Returning to the activity we started with, asking children to place the numbers onto the line allows us to revisit the concept of the midpoint and builds on our last activity. Children should have a better sense of being able to place the numbers evenly, but the main teaching point here is that they should be able to reason where their numbers sit in relation to each other.

You may ask them to compare their number line with a friend's number line, which provides an opportunity to ask which is better and why. Whose numbers are more evenly spaced? Have they correctly found the midpoint? At this stage, teachers can assess whether children are placing their number between or on the intervals.

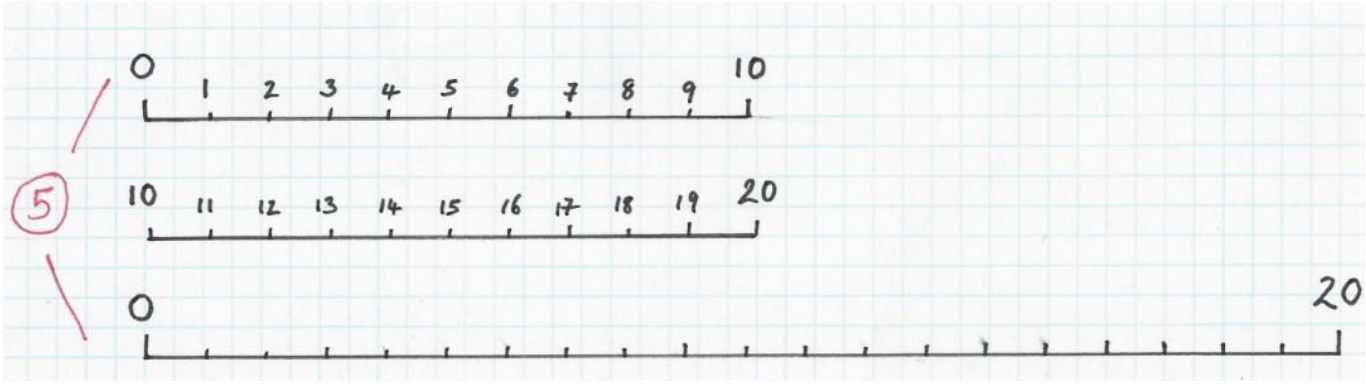
4. Using a number line as a measuring tool



This time we adjust the blank number line slightly by adding measures. Measure operates within our linear number system so a ruler is, in essence, a number line. Armed with their 10cm number line strip, can the children find objects shorter than 10cm or longer than 10cm?

By reminding children to line up their strip of paper so that 0cm lies at one end of the object, we are developing accuracy, which children will need when they start using a ruler. Can children find the midpoint? We can move on by asking them to find objects longer than 5cm or shorter than 10cm. We may even start to estimate and reason about where numbers fit in the linear system by asking where 7cm might be. Can children reason that it is between 5cm and 10cm? Where would they place it?

5. Comparing values on two completed number lines—zero to twenty



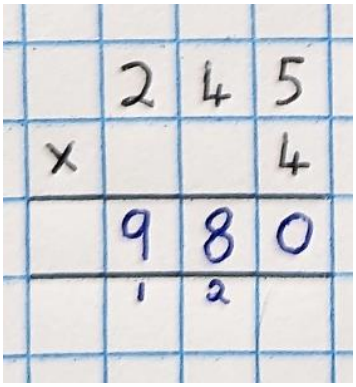
Comparing number lines encourages children to notice what is the same and what is different about numbers zero to 10 and numbers 10 to 20. With the number lines placed one on top of each other and lined up, ask children what they notice. Draw children's attention to the fact that where we have 1, on the next number line we have 10 and 1, which we call 11. Where we have 2, on the next number line we have 10 and 2, which we call 12 and so on. This enables children to see the relationship between the numbers that come after 10 and the numbers that come before 10 and the pattern of the number system.

The children can discuss what is the same and what is different about the midpoints of both number lines. Will the midpoint always feature a five? By introducing a new number line, placed underneath, with the numbers zero to 20 placed at either end, children will identify that our midpoint is now 10.

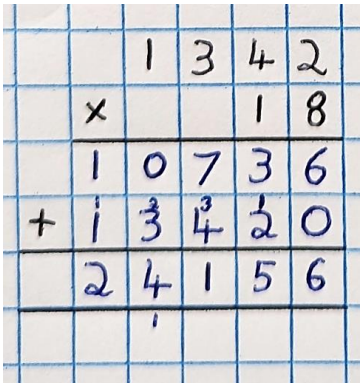
This activity enables children to explore '10 and a bit' numbers—the tricky teen numbers—and how they fit into the linear number system. This can be a 'wow' moment for children, as they start to make those connections, recognise those patterns, and begin to understand how they repeat into infinity.

Column Methods

1-digit multiplier:



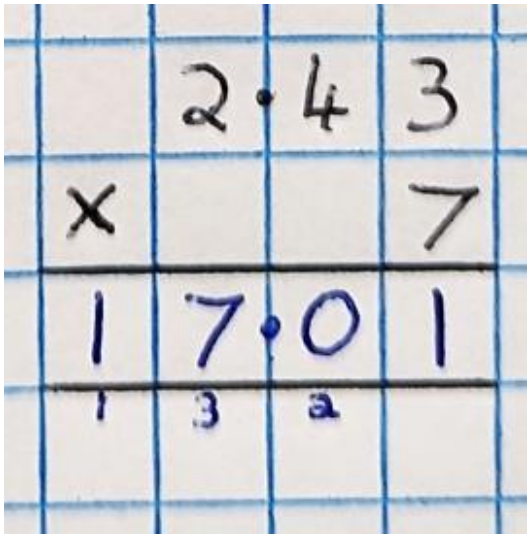
2-digit multiplier:

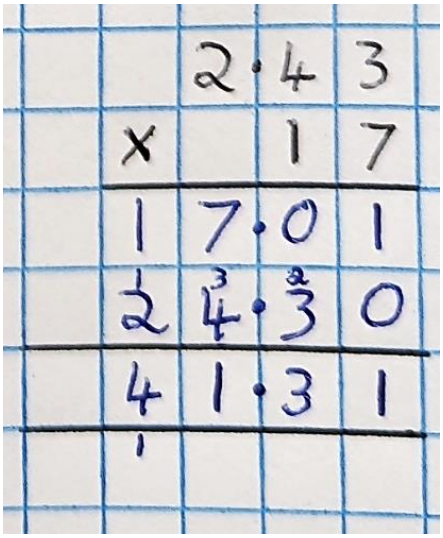


All digits are to be written with one digit per square. Ensure that the multiplicand and multiplier are written with the ones columns aligned, and that the lines are drawn with a ruler. For multiplication, as the product may be larger than the multiplicand and the multiplier, always draw the lines one extra column wide. This reinforces the idea that multiplication results in a larger product (in the case of positive integers). Ensure that children do not continue their lines past the ones column past where the decimal point would go – this is essential for avoiding misconceptions.

When writing carries (exchanges) underneath, ensure that children write very small, clear digits in the middle-top of the square in the appropriate column. This ensures that they do not confuse them with other digits in the method, and that we do not cause confusion in other methods in UKS2. A sharp pencil is essential for this. Encourage children to sharpen them habitually at the beginning of each lesson.

When using decimal numbers, the decimal point should always be placed in the centre of a line between two squares, not at the bottom and not in a square of its own on the page. This ensures that children do not treat it as an extra place value column between the ones and the tenths, and also that it is not confused with a full stop, as below. The multiplier is a whole number, however it is to be placed in line with the smallest place value column in order to provide children with a less confusing method to follow, which is procedurally logical after mastering the above method for 1-digit and 2-digit integers as multipliers. It is good practice to have children place the decimal point in all answer rows at the beginning of the calculation, to ensure that they are all lined up and no product values are accidentally multiplied by 10/100/1,000 by the omission of the decimal point.



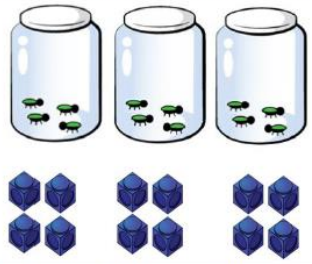
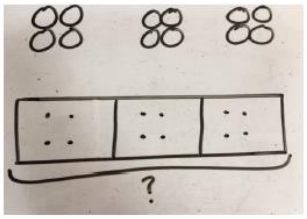
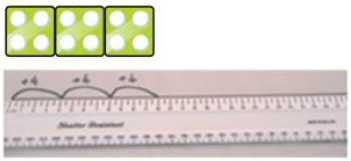
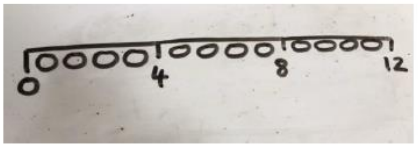
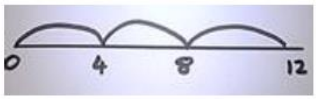
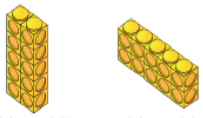
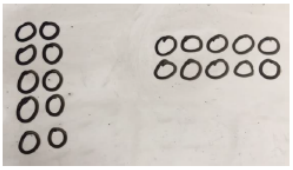
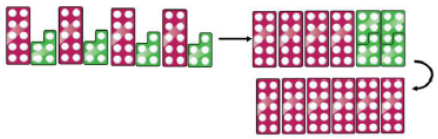
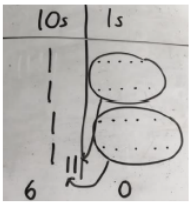
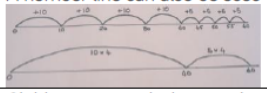
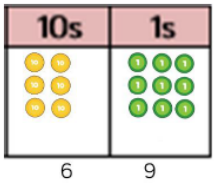
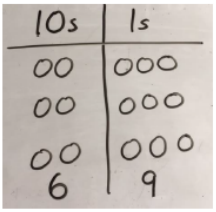


Appendix 2 – White Rose Calculation Guidance

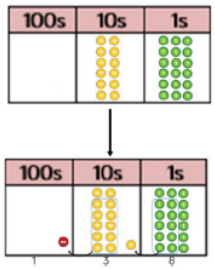
Below is a table taken from the White Rose Calculation guidance which shows how the concrete-pictorial-abstract approach can be used as a progression in learning of the concept of multiplication.

Calculation policy: Multiplication

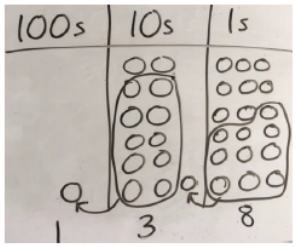
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4  Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. 3 × 4 = 12 
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 × 15 10 5 10 × 4 = 40 5 × 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3 × 23 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3 × 23 3 × 20 = 60 3 × 3 = 9 20 3 60 + 9 = 69 23 × 3 69

Formal column method with place value counters.
6 x 23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6 × 23

23	23	23	23	23	23

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

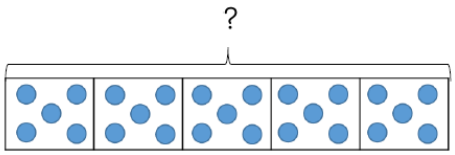
□ = 6 × 23

$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array}$$

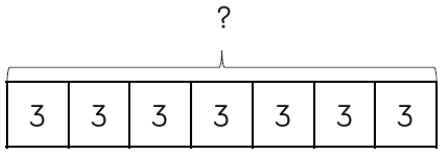
What is the calculation?
What is the product?

Appendix 3 – White Rose Guidance on the Benefits of Different Representation Methodologies

Bar Model

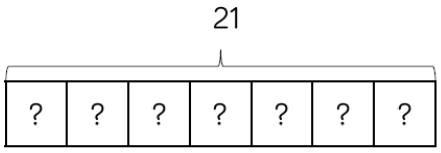


$5 \times 5 = 25$

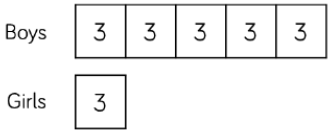


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

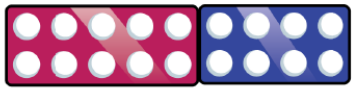
Number Shapes

$5 \times 4 = 20$

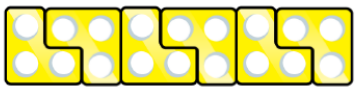
$4 \times 5 = 20$

$5 \times 4 = 20$

$4 \times 5 = 20$



$18 \div 3 = 6$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd × odd = even, odd × even = odd, even × even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

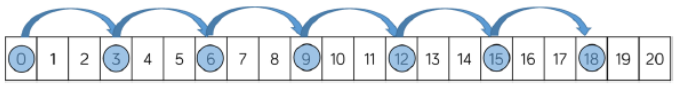
Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.
Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

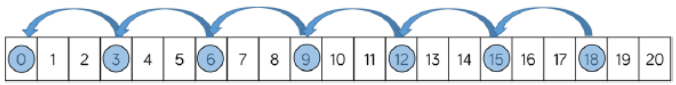
Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Tracks



$6 \times 3 = 18$
$3 \times 6 = 18$



$18 \div 3 = 6$

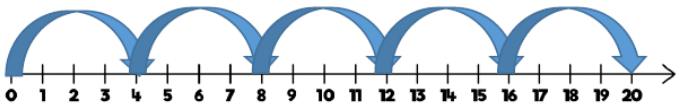
Benefits

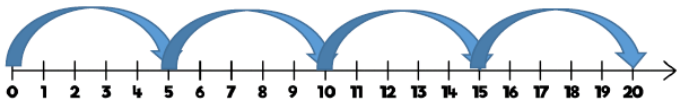
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

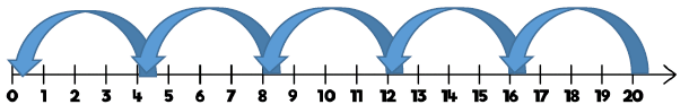
Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)





$4 \times 5 = 20$
$5 \times 4 = 20$



$20 \div 4 = 5$

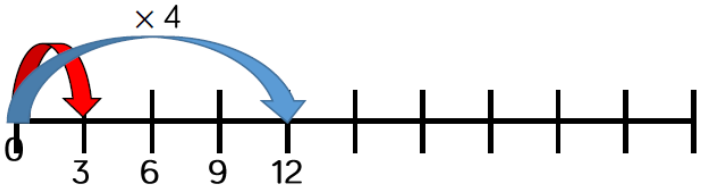
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

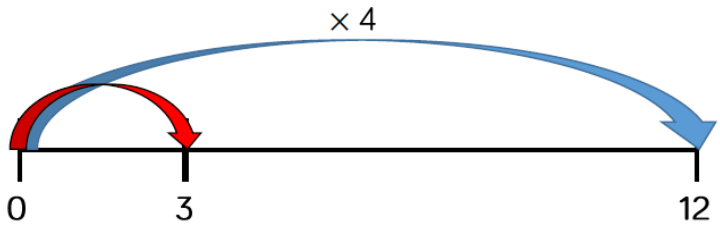
When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

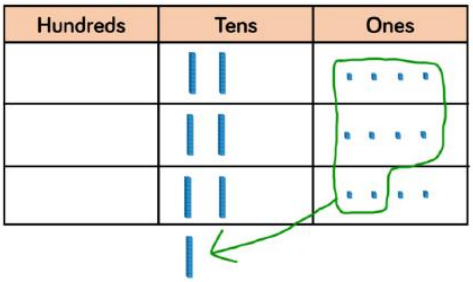
Benefits

Children can use blank number lines to represent scaling as multiplication or division.

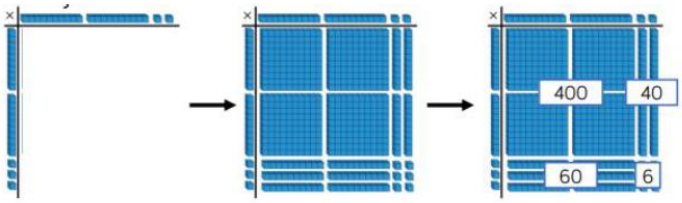
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)



$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ \hline 1 \end{array}$$



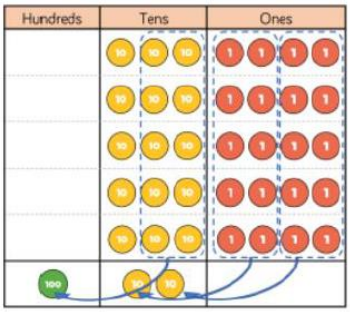
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

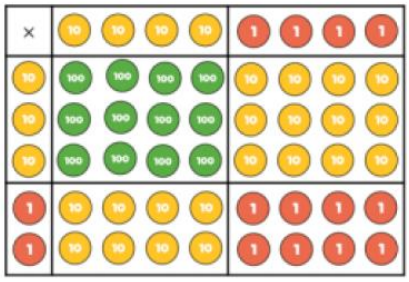
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Place Value Counters (multiplication)



$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 170 \\ \hline 1\ 2 \end{array}$$



$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Appendix 4 – The Value of Precise Mathematical Vocabulary

The following are edited extracts from an article taken from RisingStars-UK.com at https://www.risingstars-uk.com/blog/november-2015/the-value-of-precise-mathematical-vocabulary in March 2024, written by Caroline Clissold.

For a comprehensive glossary, please see the separate document "NCETM Maths Glossary KS1-KS3" which is saved in PDF format with our calculation policies in the shared area.

For **addition**, precise terms include: **augend, add, addend, equal** and **sum**.
Augend is the amount that you start off with, addend is what you add to it and sum is the result. Augend comes from the Latin augendum, a thing to be increased.
Addend comes from the Latin word addendum which is an addition made to something.
Sum comes from the Latin word summa, which means highest.

The precise terms for **subtraction** are **minuend, subtract, subtrahend, equal** and **difference**.
Minuend stems from the Latin minuendus which means to be diminished or make smaller.
Subtrahend comes from the Latin subtrahendum which means to delete from a list or take away.
The **difference**, from the Latin word differentia meaning carrying away, is the result of the subtraction.

In **multiplication** the precise terms are: **multiplicand, multiplied by, multiplier, equal** and **product**.
Multiplicand comes from the Latin word multiplicandus which means to be increased or multiplied.
Multiplier is the number you are multiplying by and product is the result of the calculation.

Dividend, divided by, divisor, equals and **quotient** are precise terms for **division**.
Dividend comes from the Latin dividendum which is an amount to be divided into groups.
Divisor is the number by which another number is divided. Its original Latin word was divider.
Quotient comes from the Latin word quotiens which means 'how many times'.
The lines around such a calculation are called the **division bracket**, which makes a lot more sense.

Commutative is another word that children need to understand and begin to use.
Commutativity is an important part of addition and multiplication. If they understand this they would only need to learn half of their number facts and multiplication tables.

In **place value** it would be worth introducing the terms **positional, multiplicative** and **additive** to help the children understand these key areas of place value. It would also help to explain that our number system increases and decreases in **powers of 10**.

When dealing with **fractions** we need to be specific about the terms used when this area is introduced to children. Children should be introduced to **numerator** and **denominator** as correct vocabulary. The line that separates the two is a **vinculum**, which in Latin means 'bond'.